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Correlations for laminar mixed convection in boundary layers adjacent to inclined, continuous moving sheets

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INTRODUCTION

THE BOUNDARY layer flow adjacent to continuous moving sheets in an otherwise quiescent ambient fluid is encountered in many industrial processes, such as material handling conveyors and extrusion of metals and plastics, etc. The heat transfer characteristics of such a process have been analyzed by several investigators (see, e.g. Moutsoglou and Chen [1] and the references cited therein), but no general and useful correlations for Nusselt numbers have been provided. In the present study, the earlier results have been extended to cover higher values of the buoyancy parameter and a wider range of Prandtl numbers. In addition, new simple correlations are presented for the local and average Nusselt numbers.

ANALYSIS

The heat transfer characteristics in the classical Blasius flow past a stationary surface and in the boundary layer flow adjacent to a continuous moving sheet in a quiescent ambient fluid are physically different. The latter case yields a higher surface heat transfer rate in comparison to the former because of the higher energy transport as a result of higher velocity in the vicinity of the moving wall.

The conservation equations and boundary conditions for boundary layers along an inclined moving sheet which is heated isothermally (UWT) or maintained at a constant surface heat flux (UHF) are the same as those stated by Moutsoglou and Chen [1, 2] and are hence not repeated here. The equations are transformed from the (x, y) primitive variables to the (ξ, η) or (ξ_1, η) coordinate system for the UWT and UHF cases, respectively. In this note, the transformed equations and boundary conditions are listed for completeness and the correlation equations for the local and average Nusselt numbers are presented, respectively, for the UWT and UHF cases.

Uniform wall temperature (UWT) case

The transformed equations and boundary conditions for mixed convection along an isothermal, continuous moving sheet inclined at an angle γ from the vertical are [1, 2]

$$f'''+\frac{1}{2}ff''\pm\xi\theta=\xi\left(f'\frac{\partial f'}{\partial\xi}-f''\frac{\partial f}{\partial\xi}\right) \quad (1)$$

$$\frac{\theta''}{Pr}+\frac{1}{2}f\theta'=\xi\left(f'\frac{\partial\theta}{\partial\xi}-\theta'\frac{\partial f}{\partial\xi}\right) \quad (2)$$

$$\begin{gathered} f(\xi,0)=0,\quad f'(\xi,0)=1,\quad \theta(\xi,0)=1 \\ f'(\xi,\infty)=0,\quad \theta(\xi,\infty)=0 \end{gathered} \quad (3)$$

where the primes denote partial differentiation with respect to η, the plus and minus signs in front of the $\xi\theta$ term in equation (1) are, respectively, for buoyancy assisting and opposing conditions, and the buoyancy parameter is given by

$$\xi(x)=Gr_x\cos\gamma/Re_x^2. \quad (4)$$

Equations (1)–(4) are valid as mentioned by Moutsoglou and Chen [2] for angles of inclination from the vertical, γ, that satisfy the condition $\tan\gamma \ll x/\delta$, which is equivalent to $\tan\gamma \ll Re_x^{1/2}/\eta_\delta$. The local Nusselt number has the expression

$$Nu_x\,Re_x^{-1/2}=-\theta'(\xi,0) \quad (5)$$

and the average Nusselt number

$$\overline{Nu}\,Re_L^{-1/2}=\xi_L^{-1/2}\int_0^{\xi_L}[-\theta'(\xi,0)]\,\xi^{-1/2}\,d\xi \quad (6)$$

where ξ_L is the buoyancy parameter based on a certain length, L, and $\overline{Nu}$ and Re_L are defined in the usual manner.

Correlation equations for the local Nusselt number in mixed convection are developed along the same line as proposed by Churchill [3]. Thus, the mixed convection local Nusselt number, Nu_x, can be written as the combination of the local Nusselt numbers, Nu_F for pure forced convection and Nu_N for pure free convection, in the form

$$Nu_x^n=Nu_F^n\pm Nu_N^n. \quad (7)$$

In this equation 'n' is a constant and the plus and minus signs pertain to buoyancy assisting and opposing situations, respectively. Equation (7) can be written in the form

$$Y^n=1\pm X^n \quad (8)$$

where

$$Y=Nu_x/Nu_F,\quad X=Nu_N/Nu_F. \quad (9)$$

It is noted that equations (8) and (9) also apply to the average Nusselt number, $\overline{Nu}$, if the local quantities Nu_x, Nu_F, and Nu_N are replaced with the corresponding average quantities, $\overline{Nu}$, $\overline{Nu}_F$, and $\overline{Nu}_N$, respectively.

The local Nusselt number for the pure forced convection in a laminar boundary layer adjacent to an isothermal, continuous moving sheet from the present calculations for $0.1 \leqslant Pr \leqslant 100$ can be expressed as

$$Nu_F=F_1(Pr)\,Re_x^{1/2} \quad (10)$$

where

$$F_1(Pr)=1.8865Pr^{13/32}-1.4447Pr^{1/3} \quad (11)$$

which is accurate to within 5%. A more general local Nusselt number expression for pure forced convection along isothermal moving sheets was also developed by taking into account the limits of $Pr\to\infty$ and $Pr\to 0$. For $Pr\to\infty$ the local Nusselt number can be expressed as

$$Nu_F=0.563Pr^{1/2}\,Re_x^{1/2} \quad (12)$$

whereas for $Pr\to 0$, the expression is given by

$$Nu_F=0.7583Pr\,Re_x^{1/2}. \quad (13)$$

Equations (12) and (13) were derived in the following manner. For very high Prandtl numbers ($Pr\to\infty$) the velocity distribution inside the thin thermal boundary layers can be approximated by a straight line ($f'(\eta)=1-C_1\eta$,

NOMENCLATURE

f, f_1 reduced stream function, $(x, y)/(\nu u_0 x)^{1/2}$

$F_1(Pr), F_2(Pr)$ functions of Prandtl number defined, respectively, by equations (11) and (17)

g gravitational acceleration

$G_1(Pr), G_2(Pr)$ functions of Prandtl number defined, respectively, by equations (27) and (33)

Gr_x local Grashof number for UWT, $g\beta(T_w - T_\infty)x^3/\nu^2$

Gr_x^* local Grashof number for UHF, $g\beta q_w x^4/k\nu^2$

Gr_L, Gr_L^* Grashof number based on L for UWT and UHF, respectively

$h, \bar{h}$ local and average heat transfer coefficients

k thermal conductivity of fluid

L a certain length of the moving sheet

n constant exponent, equations (7) and (8)

Nu_F, Nu_N, Nu_x local Nusselt numbers for pure forced, pure free, and mixed convection, hx/k

$\overline{Nu}_F, \overline{Nu}_N, \overline{Nu}$ average Nusselt numbers for pure forced, pure free, and mixed convection, $\bar{h}L/k$

Pr Prandtl number

q_w local surface heat flux

Re_x local Reynolds number, $u_0 x/\nu$

Re_L Reynolds number based on L, $u_0 L/\nu$

T fluid temperature

T_w wall temperature

T_∞ free stream temperature

u, v streamwise and normal velocity components

u_0 velocity of the moving sheet

x, y axial and normal coordinates.

Greek symbols

β volumetric coefficient of thermal expansion

γ angle of inclination from the vertical

δ boundary layer thickness

ψ stream function

η dimensionless pseudo-similarity variable $y(u_0/\nu x)^{1/2}$

ν kinematic viscosity

θ dimensionless temperature for UWT, $(T - T_\infty)/(T_w - T_\infty)$

ϕ dimensionless temperature for UHF, $(T - T_\infty)Re_x^{1/2}/(q_w x/k)$

ξ, ξ_1 buoyancy parameter for UWT and UHF defined, respectively, by equations (4) and (23).

where C_1 was found to be 0.4437). This approximation along with $\xi = 0$ was introduced into the energy equation (2) and the resulting expression for $-\theta'(0)$ was numerically evaluated to obtain the local heat transfer results for forced convection at very high Prandtl numbers. The best fit for these results yielded equation (12) which shows that for a continuous moving sheet the local Nusselt number varies as $Pr^{1/2}$, as opposed to the $Pr^{1/3}$ variation for a stationary isothermal surface, when $Pr \to \infty$. Similarly, for very low Prandtl numbers ($Pr \to 0$), the velocity distribution in the large thermal boundary layer can be approximated by the free stream velocity, namely, $f'(\eta) = 0$ or $f(\eta) = C_2$, a constant. This approximation yields a closed form expression for the local Nusselt number as $Nu_x Re_x^{-1/2} = C_2 Pr$, with $C_2 = 0.7583$, as represented by equation (13).

Thus for any given Prandtl number, a combination of equations (12) and (13) will provide an expression for the local Nusselt number in forced convection. This leads to the following expression:

$$Nu_F = F_1^*(Pr)Re_x^{1/2} \tag{14}$$

where

$$F_1^*(Pr) = 0.563Pr^{1/2}[1 + 0.712(0.02/Pr)^{1/2}]^{-3} \tag{15}$$

which is a more general form than that given by equation (10). Equation (15) was checked to give results which are accurate to within 10% in the Prandtl number range $0.01 \leqslant Pr \leqslant \infty$. For $0.1 \leqslant Pr \leqslant 100$, it agrees to within 7% of the calculated results and is thus not as accurate as equation (11).

The local Nusselt number expression for free convection along an inclined plate is given by

$$Nu_N = F_2(Pr)(Gr_x \cos\gamma)^{1/4} \tag{16}$$

where

$$F_2(Pr) = 0.75Pr^{1/2}[2.5(1 + 2Pr^{1/2} + 2Pr)]^{-1/4}. \tag{17}$$

It is noted that equation (16) is a modified form of that for a vertical plate, with $F_2(Pr)$ given by Ede [4], by simply replacing Gr_x with $Gr_x \cos\gamma$ and that pure free convection corresponds to the case when both the plate and the ambient fluid are at rest simultaneously. The local mixed convection Nusselt number for an inclined, isothermal, continuous moving sheet can then be expressed according to equation (8) as follows:

$$Nu_x Re_x^{-1/2}/F_1(Pr) = \{1 \pm [F_2(Pr)(Gr_x \cos\gamma/Re_x^2)^{1/4}/F_1(Pr)]^n\}^{1/n}. \tag{18}$$

Similarly, the corresponding average mixed convection Nusselt number can be correlated as

$$\overline{Nu}\, Re_L^{-1/2}/2F_1(Pr) = \{1 \pm [2F_2(Pr)(Gr_L \cos\gamma/Re_L^2)^{1/4}/3F_1(Pr)]^n\}^{1/n}. \tag{19}$$

Equations (18) and (19) have the form $Y = (1 \pm X^n)^{1/n}$. As will be seen later, $n = 3$ provides the best correlation. Similar correlations as given by equations (18) and (19) with $F_1(Pr)$ replaced by $F_1^*(Pr)$, can be obtained by using equations (14) and (16).

Uniform surface heat flux (UHF) case

The transformed equations and boundary conditions for mixed convection in a laminar boundary layer along an inclined, continuous moving sheet subjected to a uniform surface heat flux are given by [1, 2]

$$f_1''' + \frac{1}{2}f_1 f_1'' \pm \xi_1 \phi = \frac{3}{2}\xi_1\left(f_1'\frac{\partial f_1'}{\partial \xi_1} - f_1''\frac{\partial f_1}{\partial \xi_1}\right) \tag{20}$$

$$\frac{\phi''}{Pr} + \frac{1}{2}f_1\phi' - \frac{1}{2}f_1'\phi = \frac{3}{2}\xi_1\left(f_1'\frac{\partial \phi}{\partial \xi_1} - \phi'\frac{\partial f_1}{\partial \xi_1}\right) \tag{21}$$

$$\begin{gathered} f_1(\xi_1, 0) = 0, \quad f_1'(\xi_1, 0) = 1, \quad \phi'(\xi_1, 0) = -1 \\ f_1'(\xi_1, \infty) = 0, \quad \phi(\xi_1, \infty) = 0 \end{gathered} \tag{22}$$

where the primes again denote partial differentiation with respect to η and the buoyancy parameter is given by

$$\xi_1(x) = Gr_x^* \cos\gamma/Re_x^{5/2}. \tag{23}$$

As for the UWT case, equations (20)–(22) are valid for inclined moving sheets, with inclination angle, γ, that satisfies $\tan\gamma \ll Re_x^{1/2}/\eta_\delta$. The local and average Nusselt numbers are given, respectively, by

$$Nu_x Re_x^{-1/2} = 1/\phi(\xi_1, 0) \tag{24}$$

and

$$\overline{Nu}\, Re_L^{-1/2} = \frac{2}{3}\xi_{1L}^{-1/3}\int_0^{\xi_{1L}} [1/\phi(\xi_1, 0)]\xi_1^{-2/3}\, d\xi_1 \tag{25}$$

where ξ_{1L} is ξ_1 based on a certain length, L.

Correlations for the UHF case can be expressed in the same form as that of equations (7)–(9). For pure forced convection under the UHF case, the local Nusselt number for the moving sheet can be expressed as

$$Nu_F = G_1(Pr)\, Re_x^{1/2} \tag{26}$$

where from the present calculations for $0.1 \leqslant Pr \leqslant 100$

$$G_1(Pr) = 2.8452 Pr^{13/32} - 2.0947 Pr^{1/3} \tag{27}$$

which has an error of less than 4%. As in the UWT case, a more general expression for the local Nusselt number in forced convection along continuous moving sheets subjected to a uniform surface heat flux was developed. In the limit of $Pr \to \infty$ the local Nusselt number can be expressed by

$$Nu_F\, Re_x^{-1/2} = 0.882 Pr^{1/2} \tag{28}$$

whereas for $Pr \to 0$ the expression is given by

$$Nu_F\, Re_x^{-1/2} = 1.4275 Pr. \tag{29}$$

Thus, for any Prandtl number the local Nusselt number can be expressed by

$$Nu_F = G_1^*(Pr)\, Re_x^{1/2} \tag{30}$$

where

$$G_1^*(Pr) = 0.882 Pr^{1/2}[1 + 0.77(0.012/Pr)^{1/2}]^{-3}. \tag{31}$$

The validity of equation (31) was checked for $0.01 \leqslant Pr \leqslant \infty$ and found to give results that are accurate to within 10% of the numerically computed values. For $0.1 \leqslant Pr \leqslant 100$, it is accurate to 8% and is not as good as equation (27). The local Nusselt number for free convection is given by the correlation

$$Nu_N = G_2(Pr)(Gr_x^* \cos\gamma)^{1/5} \tag{32}$$

where

$$G_2(Pr) = Pr^{2/5}[4 + 9Pr^{1/2} + 10Pr]^{-1/5} \tag{33}$$

which is derived from that for a vertical plate given by Fujii and Fujii [5]. The local and average mixed convection Nusselt number for the inclined moving sheets can then be written according to equation (8) as follows:

$$Nu_x Re_x^{-1/2}/G_1(Pr) = \{1 \pm [G_2(Pr)(Gr_x \cos\gamma/Re_x^{5/2})^{1/5}/G_1(Pr)]^n\}^{1/n} \tag{34}$$

and

$$\overline{Nu}\, Re_L^{-1/2}/2G_1(Pr) = \{1 \pm [5G_2(Pr)(Gr_L \cos\gamma/Re_L^{5/2})^{1/5}/8G_1(Pr)]^n\}^{1/n}. \tag{35}$$

Again, equations (34) and (35) have the form $Y = (1 \pm X^n)^{1/n}$. It will also be seen later that $n = 3$ gives the best correlation. It should be noted that similar expressions for the local and average Nusselt numbers as given by equations (34) and (35), with $G_1(Pr)$ replaced by $G_1^*(Pr)$, can be developed when use is made of equations (31) and (32).

RESULTS AND DISCUSSION

The correlation equations (18) and (34) along with calculated results for Prandtl numbers of 0.7, 7 and 100 are presented in the Y vs X form for both the buoyancy assisting and opposing flow conditions, respectively, in Figs. 1 and 2 for the UWT and UHF cases. Computations were extended to higher values of the buoyancy parameter than the work of Moutsoglou and Chen [1] and to cover an additional Prandtl number of 100. As is evident from the figures, an exponent value of $n = 3$ correlates very well (with errors of less than 5%) for both the heating conditions and for both the buoyancy assisting and opposing situations. The average Nusselt numbers as calculated by equations (6) and (25) for the UWT and UHF cases, respectively, were then correlated with the respective equations (19) and (35), and good agreement was found to exist between the calculated and the correlated results. Thus, separate figures for the average Nusselt number correlations are not presented. Instead, Figs. 1 and 2 may be utilized for this purpose provided the Y and X coordinates in these figures are represented by those given in equations (19) and (35).

CONCLUSION

Simple and accurate correlation equations have been developed and presented for estimating the local and average Nusselt numbers in mixed convection adjacent to inclined,

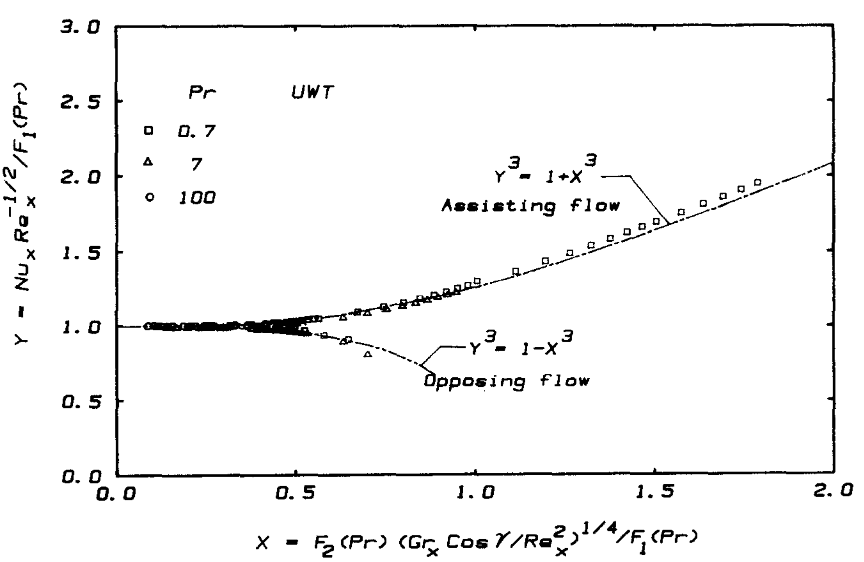

FIG. 1. A comparison between the predicted and correlated local Nusselt numbers for the UWT case.

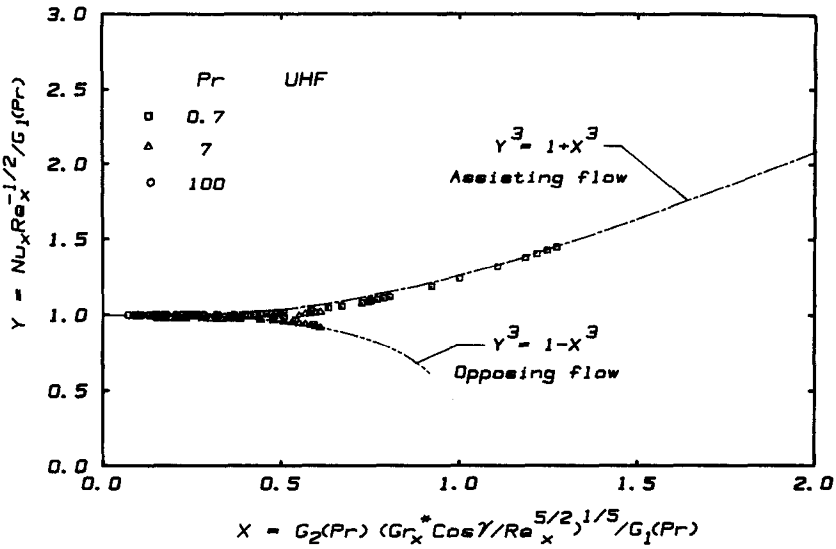

FIG. 2. A comparison between the predicted and correlated local Nusselt numbers for the UHF case.

continuous moving sheets the surfaces of which are maintained either at a constant temperature or at a constant heat flux. The correlations presented for a Prandtl number range of $0.7 \leqslant Pr \leqslant 100$ and for both buoyancy assisting and opposing flow conditions agree very well with the analytically predicted values.

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REFERENCES

1. A. Moutsoglou and T. S. Chen, Buoyancy effects in boundary layers on inclined, continuous moving sheets, *ASME J. Heat Transfer* **102**, 371–373 (1980).
2. A. Moutsoglou and T. S. Chen, Mixed convection on inclined surfaces, *ASME J. Heat Transfer* **101**, 422–426 (1979).
3. S. W. Churchill, A comprehensive correlating equation for laminar assisting, forced and free convection, *A.I.Ch.E. Jl* **23**, 10–16 (1977).
4. A. J. Ede, Advances in free convection. In *Advances in Heat Transfer*, Vol. 4, pp. 1–64. Academic Press, New York (1967).
5. T. Fujii and M. Fujii, The dependence of local Nusselt number on Prandtl number in the case of free convection along a vertical surface with uniform heat flux, *Int. J. Heat Mass Transfer* **19**, 121–122 (1976).

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Correction de longueur d'impulsion pour la mesure de la diffusivité thermique par méthode flash

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Il s'agit d'étendre le résultat de Azumi et Takahashi [1] obtenu dans le cas d'un échantillon supposé isolé (Parker *et al.* [2]), au cas général avec pertes thermiques. La solution analytique de l'équation de diffusion permettant de calculer la température sur la face opposée à l'impulsion de flux est donnée par le produit de convolution suivant

$$T(t) = Q\int_0^t \varphi(\tau)T_\Delta(t-\tau)\,d\tau$$

où $T_\Delta(t)$ est la solution pour une impulsion de Dirac, $\varphi(t)$ la forme de l'impulsion avec

$$\int_0^\infty \varphi(t)\,dt = 1$$

et Q l'énergie de l'impulsion.

De façon générale, la solution à l'impulsion de Dirac en présence de pertes s'écrit sous la forme d'une double série [3]

$$T_\Delta(t) = \sum_n\sum_p A_{np}\exp(-\nu_{np}t).$$